Assignment 1 - Optimization Method

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1. If is a convex combination of , then we can get a system with three equations.

Solve this system, we can get . Therefore, .

1. Function is convex, it satisfies

When it comes to , the left-hand side of this inequality can be rewritten as:

. And the right -hand side of this inequality can be rewritten as:

If , the left-hand side of the inequality must be less than or equal to right.

If , the left-hand side of the inequality must be larger than or equal to right.

1. We know that is a concave function, is a convex function. Both of them depend on . where is a positive-valued constant, and condition is on the set .

For

For

Set , . .

. Then we can analyze the result: since g(x) is concave, . , since is a convex function, . Thus, is convex function in this case.

1. We know that . Then proved by:

Hence, for

( here )

1. (a) We know that . And we should transform to the form , For matrix , the diagnose of is the quadratic coefficient, and other is the half of the coefficient of the first degree.

So ,

(b)

(c)

(d) Calculate the determinant of .. Thus,

.

Solve this equality and we can get . All of the eigenvalue are larger than zero, so is positive semi-definite matrix which means that is convex function.

1. We know that

Since , .

Since ,

Thus, .

(b) When , . Since , is not a singular matrix, which means that it must exist solution in this case.

1. We know that , we should calculate the determinant of .

, so

Simplify the equality, . According to Square root formula of quadratic equation with one unknown, .

Therefore, , . If both of them are larger than 0, is the positive definite matrix and the positive semi-definite matrix. Since , must be larger than 0. For , if , . If , . To conclude, when , A is the positive definite matrix and the positive semi-definite matrix. When , A is positive semi-definite matrix.